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DEVELOPMENT OF THREE-DIMENSIONAL PERTURBATIONS IN RAYLEIGH–TAYLOR INSTABILITIES

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The study of the Rayleigh–Taylor instability (RTI) is a quite relevant problem. Besides the theoretical interest, it is valuable for a number of important practical problems, such as stability studies of shell compression in problems of laser thermonuclear synthesis, obtaining superstrong magnetic fields, etc.

The following clearly expressed stages can be traced in the evolution of RTI: linear, intermediate, regular asymptotic, and turbulent [1, 2]. The linear stage is characterized by a small amplitude a in comparison with the perturbation wavelength L and an exponential velocity growth. When the perturbation amplitude a reaches 0.4L, the process evolves to a stage intermediate between the linear and the regular asymptotic one. At the regular asymptotic stage, starting at $a \approx 0.75L$, heavy fluid "peaks" are definitely formed, breaking down with constant acceleration, as well as light fluid "bubbles," floating with constant velocity. This RTI is unstable [1, 2] and changes into the turbulent stage, during which there is intense interaction of various wavelength perturbations and fluid mixing.

The RTI was investigated in most detail for a planar surface section and a ratio of heavy to light fluid densities tending to infinity. The linear stage was studied in classical papers [3-5], the regular asymptotic stage in [6-8], a phenomenological theory of the turbulent stage was developed in [9], and a discussion of the mechanism of its formation was given in [2].

An analytical mathematical apparatus for analysis is hardly available, however, since experimental studies of RTI are quite difficult. The most complete information can be obtained from numerical calculations; thus, the case of a free surface was investigated in [10], that of two incompressible fluids in [11], and that of two compressible media in [12]. We also point out [13], where numerical calculations of RTI of a compressible shell were performed.

So far only the two-dimensional case was considered both analytically and computationally. The two-dimensional model is, however, physically inadequate: in a physical experiment the two-dimensional structures are destroyed by the transverse shortwave instability, transforming into three-dimensional ones.

The numerical methods used in [10, 11] can be extended, in principle, to the three-dimensional case. This leads, however, to a quite significant increase in the computing time and an increase in the required computer memory, so that detailed calculations cannot be realized on contemporary computers.

In the present paper we perform a numerical experiment on three-dimensional RTI by means of the coarse particle method (see, e.g., [14]), widely recommended in solving a wide class of complex problems of gas hydrodynamics (see, e.g., [14, 15]). The development of two-dimensional RTI up to large amplitudes, when the process becomes substantially nonlinear, was first investigated by the given method in [12].

The full spatial three-dimensional nonstationary system of vortex Euler equations with account of a gravitational field is solved by the coarse particle method

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 117-122, January-February, 1981. Original article submitted September 17, 1979.

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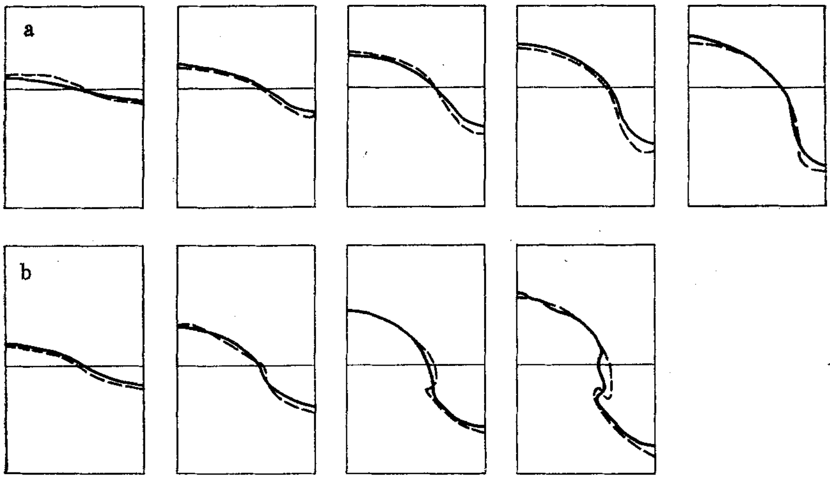

Fig. 1

$$\begin{aligned}
&\partial\rho/\partial t + \operatorname{div}(\rho \mathbf{W}) = 0, \\
&\partial\rho u/\partial t + \operatorname{div}(\rho u \mathbf{W}) + \partial p/\partial x = 0, \\
&\partial\rho v/\partial t + \operatorname{div}(\rho v \mathbf{W}) + \partial p/\partial y + \rho g = 0, \\
&\partial\rho w/\partial t + \operatorname{div}(\rho w \mathbf{W}) + \partial p/\partial z = 0, \\
&\partial\rho E/\partial t + \operatorname{div}(\rho E \mathbf{W}) + \operatorname{div}(p\mathbf{W}) + \rho g v = 0,
\end{aligned} \qquad (1)$$

where $W = \{u, v, w\}$ is the velocity vector; ρ, medium density; $E = e + W^2/2$, specific total energy; e, specific internal energy; p, pressure; and the free-fall acceleration g is directed along the y axis.

The programming of the given numerical experiment was carried out within the package of the applied KRUCHA (coarse particle) programs [16], to which modules reflecting the specific problem (gravitation, initial data) were added. We point out that though as a closure equation of system (1), one uses the equation of state of an ideal gas $e = p/(\varkappa - 1)\rho$ ($\varkappa$ is the adiabatic index), the compressibility has little effect on the nature of the process (the Mach number value in the calculation region never exceeds 0.3).

The calculation is straightforward without separating the contact surface of the heavy and light fluids. The two-dimensional calculations have shown that despite the "smearing" of the contact surface of separation (7-8 cells in the "peak" region, and 4-5 cells in the "bubble" region), it can be identified with sufficient accuracy with a surface on which $\rho = 0.5 \times (\rho_1 + \rho_2)$, where ρ_2 is the heavy fluid density and ρ_1 is that of the light one (a definition of this surface by other heuristic properties, such as max grad ρ, etc., gives closely related results). Two-dimensional test problems were calculated in order to compare with data of other authors by using the three-dimensional program. The initial data and test parameters were identical to those used in [11], except that the medium was treated with the equation of state of an ideal gas. Due to the smallness of the Mach number in the range of calculation, the compressibility is low in the present case, and therefore a comparison is possible with the calculated flow of an incompressible fluid. Below we present some of the results obtained. The sizes of the calculating regions were 20 cells along the horizontal x axis and 60 along the vertical y axis. At all boundaries symmetry conditions were imposed, so the mass, momentum, and energy flows at the region boundaries vanished. Since at the upper and lower boundaries $v \equiv 0$, we have by the equation for the vertical momentum component $\partial p/\partial y + \rho g = 0$. This boundary condition was imposed on the upper and lower boundaries to eliminate the possibility of perturbation generation on them. The initial data were assigned at the moment of time $t = 0$: in the upper half-space we have the heavy fluid with density ρ_2, and in the lower half the light one with density $\rho_1 = 0.1$. On the medium boundaries $e_2/(\varkappa - 1) = 10$, $e_1/(\varkappa - 1) = 100$, and in both media $\partial e/\partial y = -g$. Thus, at the initial moment $\partial p/\partial y + \rho g = 0$, and the evolution of instability occurs from the equilibrium position. The initial perturbation was given in the form of a velocity field:

$$u = A \sin\frac{\pi x}{L}\,[2H(y) - 1]\exp\left(\frac{-\pi\,|y|}{L}\right),$$

$$v = A\cos\frac{\pi x}{L}\exp\left(\frac{-\pi\,|y|}{L}\right),$$

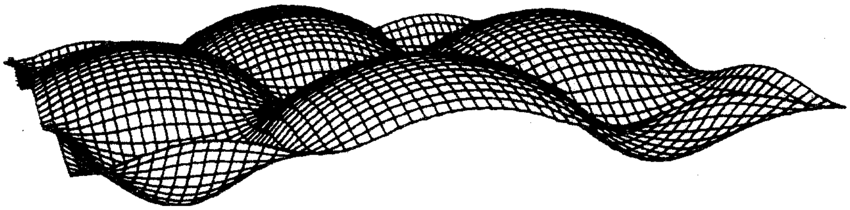

Fig. 2

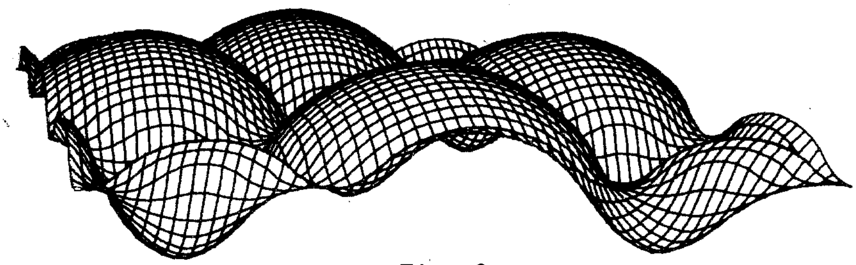

Fig. 3

where H(y) is the Heaviside function

$$H(y)=\begin{cases}1 & y>0,\\ 0 & y<0,\end{cases}$$

and L is the transverse size of the region. In units made dimensionless L = 2, g = 1, A = 0.78, $\delta x = \delta y = 0.1$. Here δx and δy are the sizes of the calculation cells along the x and y axes, respectively. We note that for a given initial perturbation in calculations with compressible media, it is possible to satisfy the condition div **W** = 0 everywhere except the surface of separation. Otherwise, density perturbations occur which can affect the pattern of RTI development.

Two-dimensional calculations by the method of coarse particles showed good agreement with [11]. Figure 1a shows the shape of the contact surface for $\rho_2/\rho_1 = 10$ at different moments of time (0.25; 0.49; 0.73; 0.97; 1.21); the solid lines correspond to calculations by the method of coarse particles, and the dashed lines are the calculations of [11]. The plot of velocity motion of the "peak" is also near that given in [11], and the mean "peak" acceleration, as in [11], is approximately the total acceleration of the gravity force. The ratio of the "bubble" radius of curvature to the perturbation wavelength R/2L was 0.40, which is closer to the value of 0.39 given in [10] and to the theoretical value 0.35 [6] than 0.48 in [11]. The rise velocity of the "bubble" is somewhat higher than in [11]. The value obtained by us is $u_B\left(\frac{\rho_2-\rho_1}{\rho_2}gR\right)^{-1/2} = 0.44$, that of [11] is 0.32, and the theoretical value for this quantity is ≈ 0.5 [7].

We note that despite the "smearing" of the contact surface, the method of coarse particles also makes it possible to follow the evolution of Kelvin–Helmholtz instability for small values of ρ_2/ρ_1. Figure 1b shows the form of the contact surface for $\rho_2/\rho_1 = 2$ at the moments of time 0.45; 0.89; 1.34; 1.79.

We turn now to consider results of numerical modeling of three-dimensional RTI by the method of coarse particles. The sizes of the calculated regions were 20, 60, and 20 cells along the x, y, and z axes, respectively. In this case the mesh parameters were $\delta x = \delta y = \delta z$. In the three-dimensional case the perturbation surface of separation can be formed either by hexagonal or quadratic lattices [17]. In the present paper we selected the latter case (there is no difficulty in modeling a hexagonal lattice). For its realization the initial perturbation was given in the form

$$u = A\sin\frac{\pi x}{L}\cos\frac{\pi z}{L}\,[2H(y)-1]\exp\left(\frac{-2\pi\,|y|}{L}\right),$$

$$v = A\cos\frac{\pi x}{L}\cos\frac{\pi z}{L}\exp\left(\frac{-2\pi\,|y|}{L}\right),$$

$$w = A\cos\frac{\pi x}{L}\sin\frac{\pi z}{L}\,[2H(y)-1]\exp\left(\frac{-2\pi\,|y|}{L}\right).$$

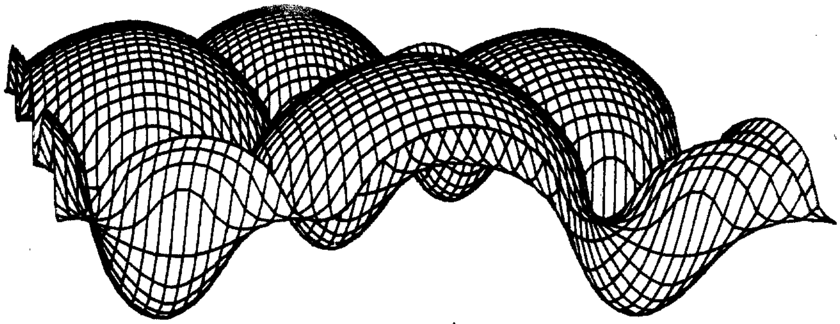

Fig. 4

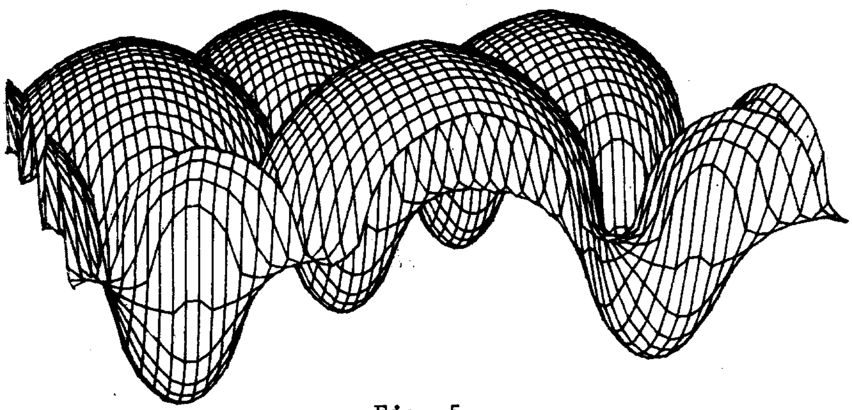

Fig. 5

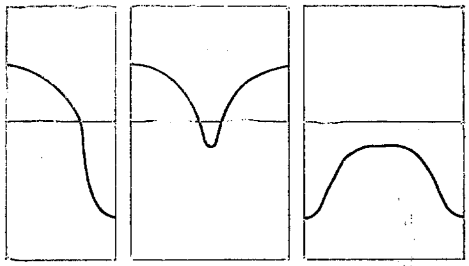

Fig. 6

In the remaining discussion all initial, boundary conditions, and problem parameters were the same as in the two-dimensional case.

Figures 2-5 give the surfaces of separation in perspective projection, constructed at the successive moments of time $t = 0.6$; 0.9; 1.2; 1.5, respectively. Clearly seen are the "buoyancy" of wide "bubbles" and the "breakdown" of sufficiently narrow "peaks."

We note that the "high" (sizes on the y azis) "bubbles" and "peaks" in Figs. 2-5 are indeed nonunique. This is explained by the fact that the point of the separation surface with coordinates $x = z = \pi/2L$, $y = 0$, at which initially $u = v = w = 0$, moves below with an acceleration approximately equal to half the "peak" acceleration. A more precise idea on the nature of the process is given by Fig. 6, in which are shown the cross sections of the contact surfaces in the planes $x = 0$ ($z = 0$) and $x = z$, $x = -z$ at the moment of time $t = 1.5$.

The ratio of "bubble" radius of curvature to the perturbation wavelength $R/2L$ is 0.34 at the cross sections $x = 0$ and $z = 0$, and 0.31 at the cross section $x = z$. The theoretical and experimental values of this quantity, obtained in studying the rise of air bubbles in vertical tubes filled by a liquid, are 0.35 [18]. The dimensionless velocity of "bubble" rising $u_B(2Lg)^{-1/2}$ was 0.31 ± 0.02, while according to the data of [18], for cylindrical tubes it is 0.32, while for tubes of rectangular cross section with lateral side ratio 1:4 it is 0.29. The "peak" acceleration was ~20% larger than in the two-dimensional case, while the "peak" itself was somewhat wider.

The authors are grateful to S. I. Anisimov for useful discussions.

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